RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2013

SECOND YEAR

Date : 23/12/2013 **MATHEMATICS (General)**

Time : 11 am – 2 pm Paper : III Full Marks : 75

[Use a separate answer book for each group]

Group - A

Answer **any four** questions :

1. Perpendiculars PL, PM, PN are drawn from the point P(a,b,c) on the coordinate planes. Show that the equation of the plane LMN is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$. [5]
2. Find the shortest distance between z-axis and the line $\frac{x-a}{\ell}=\frac{y-b}{m}=\frac{z}{n}$. [5]
3. Find the image of the point (2,6,–4) on the plane $x-5y+2z+6=0$. [5]
4. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact. [2+3]
5. If every plane cuts a quadric surface in a circle, show that the surface is a sphere. [5]
6. Find the equation of the right circular cone whose vertex is the origin, axis is the x-axis and semi-vertical angle is 60°. [5]

Group - B

7. Answer **any four** questions : [4×2]
   a) What is Basic Solution for a system of equations.
   b) State the Fundamental theorem of L.P.P.
   c) Show that the set $S=\{(x,y)/x\le 5, y\ge 3\}$ is a convex set.
   d) Find a basic feasible solution of the system of equations
   $x_1+2x_3=1$
   $x_2+x_3=4$
   e) Determine the extreme points of the set $x=\{(x,y): x^2+y^2\le 25\}$
   f) Define optimal solution of an L.P.P. What is the difference between feasible solution and optimal solution.
   g) Introduce slack and surplus variables to convert the following L.P.P into standard form :
   Maximize $Z = 2x_1 + x_2$
   Subject to $x_1 \le 4$
   $2x_1 + x_2 \ge 1$
   and $x_1, x_2 \ge 0$

Answer **any two** questions :

8. a) A factory engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and those for one unit of B are 3, 1 and 3 hours respectively. The profit on each unit of A and B are Rs. 2·00 and Rs. 3·00 respectively. Assuming that 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembling time are available, pose an L.P.P in terms of maximizing the profit of the item manufactured. [4]

b) Reduce the feasible solution (1,1,1) of the set of equations

$x_1 + x_2 + 2x_3 = 4$

$2x_1 - x_2 + x_3 = 2$

to a basic feasible solution. [6]

c) Solve graphically the following L.P.P.

Maximize $Z = 2x_1 + 4x_2$

Subject to $x_1 + 2x_2 \le 5$

$x_1 + x_2 \le 4$

and $x_1, x_2 \ge 0$ [6]

9. a) Prove that the set of all feasible solutions of an L.P.P is a convex set. [4]

b) Use simplex method to solve the L.P.P.

Maximize $Z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

and $x_1, x_2 \ge 0$ [6]

c) Obtain an initial B.F.S. to the following transportation problem using VAM method. [6]

	D_1	D_2	D_3	D_4	
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
	6	10	12	15	

10. a) Prove that the dual of the dual is the primal. [8]

b) Solve the following L.P.P by Charne's Big-M Method :

Minimize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \ge 8$

$3x_1 + 2x_2 \ge 12$

$x_1, x_2 \ge 0$ [8]

11. a) If x_1, x_2 be reals, show that the set given by $X = \{(x_1, x_2) : x_1 + 2x_2 = 5\}$ is a convex set. [5]

b) Formulate the dual of the L.P.P. :

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \le 3$

$3x_2 - x_3 \ge 5$

$x_1, x_2 \ge 0$ and

x_3 is unrestricted in sign. [5]

c) Solve the assignment problem given by the following cost-matrix : [6]

	a	b	c	d
1	18	17	12	11
2	19	15	11	16
3	25	21	17	11
4	16	14	11	11

Group – C

Answer **any three** questions :

12. a) Round-off the following numbers to three significant digits: 0·999, –239·85, 0·0056812 [3]
    b) Show that $\Delta.\nabla f(x) = (\Delta - \nabla)f(x)$, under usual notations. [2]

13. Given the following table :

x :	1·00	1·10	1·20	1·30
f(x):	0·8415	0·8912	0·9320	0·9636

    Construct the difference table and compute f(1·02) by Newton's Forward Formula. [5]

14. Evaluate $\int_0^1 \frac{dx}{1+x^2}$, taking 5 sub-intervals, by Trapezoidal Rule, correct to five significant figures. Also compute the Relative Error. [4+1]

15. Compute one positive root of $2x - 3\sin x - 5 = 0$ by the bisection method, correct to three significant figures. [5]

16. Find a positive root of $x + \log_e x - 2 = 0$, by Newton – Raphson Method, correct to six decimal places. [5]